TRANSPORTNE LASTNOSTI

19. BOLTZMANNOVA ENAČBA

10.

Elektrone v kristalu obravnavamo kot plin neodvisnih delcev, ki trkajo med seboj, kot v kinetični teoriji plinov.

Delamo kvaziklasično, valovni paketi z $\vec{r}(t)$, $\vec{k}(t)$, znotraj enega pasu.
Za motnje velja $q \ll k_F$, $\hbar\omega \ll \varepsilon_F$.

Porazdelitvena funkcija $f(\vec{r},\vec{k},t)$, $dN = f(\vec{r},\vec{k},t)\, d^3\vec{r}\, \frac{d^3\vec{k}}{(2\pi)^3}\cdot 2$ ↑spin

Gostota $n(\vec{r},t) = 2\int \frac{d^3\vec{k}}{(2\pi)^3} f(\vec{r},\vec{k},t)$

Gostota toka $\vec{j}_n(\vec{r}) = 2\int \frac{d^3\vec{k}}{(2\pi)^3}\, \vec{v}_k\, f(\vec{r},\vec{k},t)$

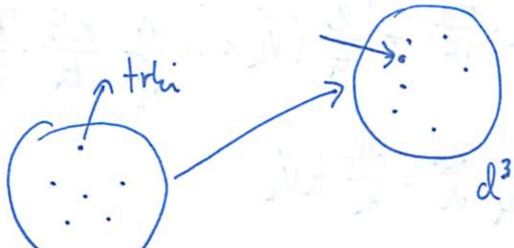
Gostota el. toka $\vec{j}_e = e\,\vec{j}_n$.

Energijski tok $\vec{j}_u(\vec{r}) = 2\int \frac{d^3\vec{k}}{(2\pi)^3}\, \varepsilon_k\, f(\vec{r},\vec{k},t)\cdot \vec{v}_k$

$$dU = TdS + \mu dN = dQ + \mu dN$$

Toplotni tok $\vec{j}_Q(\vec{r}) = \vec{j}_u - \mu\vec{j}_n = 2\int \frac{d^3\vec{k}}{(2\pi)^3}\,(\varepsilon_k-\mu)\,\vec{v}_k\, f(\vec{r},\vec{k},t)$

Sledimo delcem v diferencialnem volumnu faznega prostora:



$dN = dN'$ če ni trkov

$d^3\vec{r}\,' d^3\vec{k}\,'(t+dt)$

$d^3\vec{r}\, d^3\vec{k}\,(t)$

$\vec{r}\,' = \vec{r} + \vec{v}\,dt = \vec{r} + \frac{1}{\hbar}\frac{\partial H}{\partial \vec{k}}\,dt \qquad \vec{k}\,' = \vec{k} + \frac{1}{\hbar}\vec{F}\,dt$

$$d^3\vec{r}\,' d^3\vec{k}\,' = J\, d^3\vec{r}\, d^3\vec{k} \qquad J = \begin{vmatrix} \frac{\partial \vec{r}\,'}{\partial \vec{r}} & \frac{\partial \vec{r}\,'}{\partial \vec{k}} \\ \frac{\partial \vec{k}\,'}{\partial \vec{r}} & \frac{\partial \vec{k}\,'}{\partial \vec{k}} \end{vmatrix} = \begin{vmatrix} 1+\frac{\partial^2 H}{\hbar\,\partial\vec{r}\,\partial\vec{k}}dt & \frac{\partial^2 H}{\hbar\,\partial\vec{k}^2}dt \\ -\frac{\partial^2 H}{\hbar\,\partial\vec{r}^2}dt & 1-\frac{\partial^2 H}{\hbar\,\partial\vec{r}\,\partial\vec{k}}dt \end{vmatrix}$$

$$J = 1 + \mathcal{O}(dt^2)$$

Liouvillova enačba:

$$f(\vec{r},\vec{k},t) = f(\vec{r}+\dot{\vec{r}}\,dt,\ \vec{k}+\dot{\vec{k}}\,dt,\ t+dt) \quad \text{oz.} \quad \frac{df}{dt} = 0$$

kjer (substancialni odvod) $\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \vec{r}}\frac{\partial \vec{r}}{\partial t} + \frac{\partial f}{\partial \vec{k}}\frac{\partial \vec{k}}{\partial t}$

Delci lahko priletijo v diferencialni volumen ali odletijo ven zaradi trkov:

$$\frac{df}{dt} = I[f]$$

$\uparrow$ trkalni člen (angl. collision integral)

BOLTZMANNOVA ENAČBA:

$$\frac{\partial f}{\partial t} + \vec{v}_k \cdot \frac{\partial f}{\partial \vec{r}} + \frac{1}{\hbar} \vec{F} \cdot \frac{\partial f}{\partial \vec{k}} = I[f]$$

V ravnovesju $f = f_0(\varepsilon)$ $\qquad f_0(\varepsilon) = \dfrac{1}{e^{\beta(\varepsilon-\mu)}+1}$ $\qquad \beta = \dfrac{1}{k_B T}$

Lokalno ravnovesje $f = f_0(\varepsilon_k, \vec{r})$ $\qquad$ z $\mu = \mu(\vec{r})$, $T = T(\vec{r})$

Majhna odstopanja: $f = f_0(\varepsilon_k, \vec{r}) + g(\vec{r}, k, t)$

$$\frac{\partial f_0}{\partial \vec{r}} = \frac{\partial f_0}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial x}\frac{\partial x}{\partial \vec{r}} = \frac{\partial f_0}{\partial \varepsilon} k_B T \left(-\frac{1}{k_B T^2} \nabla T (\varepsilon - \mu) + \frac{1}{k_B T}(-\nabla \mu) \right)$$

$x = \beta(\varepsilon - \mu) = \dfrac{\varepsilon - \mu}{k_B T}$ $\qquad$ torej $\dfrac{\partial f_0}{\partial \vec{r}} = -\dfrac{\partial f_0}{\partial \varepsilon} \dfrac{\nabla T(\varepsilon - \mu) + T \nabla \mu}{T}$

$$\frac{\partial g}{\partial t} - \vec{v}_k \cdot \frac{(\varepsilon_k - \mu)\nabla T + T\nabla\mu}{T}\frac{\partial f_0}{\partial \varepsilon} + \vec{v}_k \cdot \frac{\partial g}{\partial \vec{r}} + \frac{q}{\hbar}(\vec{v}_k \times \vec{B})\frac{\partial g}{\partial \vec{k}} + \frac{q\vec{E}}{\hbar}\cdot\frac{\partial f_0}{\partial \vec{k}} = I[f_0 + g]$$

a) $\dfrac{\partial f_0}{\partial \vec{k}} \cdot (\vec{v}_k \times \vec{B}) = 0$ ker $\dfrac{\partial f_0}{\partial \vec{k}} = \dfrac{\partial f_0}{\partial \varepsilon_k}\dfrac{\partial \varepsilon_k}{\partial \vec{k}} = \dfrac{\partial f_0}{\partial \varepsilon} \hbar \vec{v}_k$

b) Izpustili smo člen drugega reda $\propto E g$.

c) Magnetno polje je lahko močno.

LINEARIZIRANA BOLTZMANNOVA ENAČBA:

$$\frac{\partial g}{\partial t} + \vec{v}_k \cdot \frac{\partial g}{\partial \vec{r}} + \frac{q}{\hbar}(\vec{v}_k \times \vec{B}) \cdot \frac{\partial g}{\partial \vec{k}} + \frac{\partial f_0}{\partial \varepsilon} \vec{v}_k \cdot \tilde{\vec{F}} = I[f_0 + g]$$

$$\vec{F} = q\vec{E} - \nabla\mu - (\varepsilon_k - \mu)\frac{\nabla T}{T}$$
$$= q\vec{\mathcal{E}} - (\varepsilon_k - \mu)\frac{\nabla T}{T}$$

$\vec{E} = -\nabla\varphi$ $\qquad \eta = \mu + q\varphi$ elektrokemijski potencial (to meri voltmeter).

$\vec{\mathcal{E}} = -\nabla \dfrac{\eta}{q}$

POZOR: različne nomenklature

2.16. SIPALNI MEHANIZMI

Naključni procesi, verjetnost za preskok na časovno enoto, $w(\vec{k}\to\vec{k}')$.

$$I[f] = \sum_{\vec{k}'} w(\vec{k}'\to\vec{k})\, f(\vec{k}')[1-f(\vec{k})] - w(\vec{k}\to\vec{k}')\, f(\vec{k})[1-f(\vec{k}')]$$

V primeru elastičnega sipanja $w(\vec{k}\to\vec{k}') = w(\vec{k}'\to\vec{k}) \equiv w(\vec{k},\vec{k}')$
(detajlno ravnovesje ali načelo mikroskopske reverzibilnosti)

Tedaj je $I[f] = \sum_{k'} w(\vec{k},\vec{k}')\,[f(\vec{k}')-f(\vec{k})]$

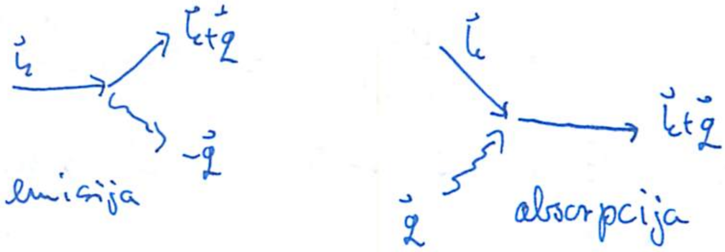
a) Sipanje na nečistočah

$$w(\vec{k}\to\vec{k}') = N_{imp}\frac{2\pi}{\hbar}\,\delta(\varepsilon_{k'}-\varepsilon_{k})\,|\langle \vec{k}'|u|\vec{k}\rangle|^2 \qquad u = \frac{Ze_0 e^{-\lambda r}}{4\pi\varepsilon\varepsilon_0 r}$$

$\rho(T) = $ konst. Rezidualna upornost pri $T=0$.

b) Sipanje na fononih

$$w(\vec{k}\to\vec{k}+\vec{q}) = \frac{2\pi}{\hbar}|M_{\vec{k}\vec{q}}|^2\left[(1+n_{-\vec{q}})\,\delta(\varepsilon_{\vec{k}+\vec{q}}+\hbar\omega_{\vec{q}}-\varepsilon_{\vec{k}}) + n_{\vec{q}}\,\delta(\varepsilon_{\vec{k}+\vec{q}}-\hbar\omega_{\vec{q}}-\varepsilon_{\vec{k}})\right]$$



emisija absorpcija

Bloch-Grüneisen:

$$\rho(T) \propto \begin{cases} T^5 & T \ll T_D \\ T & T \gg T_D \end{cases}$$

c) Sipanje e^- na e^-

$\rho(T) \propto T^2$ (obravnavamo v poglavju o Fermijevih tekočinah

3.~~1~~1. PRIBLIŽEK RELAKSACIJSKEGA ČASA

Transportni relaksacijski čas:

$$I[f_0+g] = -\frac{g}{\tau} \qquad\qquad f = f_0 + g(t=0)\,e^{-t/\tau}$$

To je možno, če je sipalni člen sorazmeren z odstopanjem g s predfaktorjem, ki ni odvisen od motnje.

POVEJ PREJ

Predpostavimo elastično sipanje. Tudi če pride do absorpcije večje energije, je zelo verjetno, da se bo ta takoj zatem ponovno izsevala. To je ekvivalentno elastičnemu sipanju v procesu drugega reda z virtualnim vmesnim stanjem. Zato je elastično sipanje dober približek (oz. dober efektivni opis).

Naj velja $g(\vec{k}) = A(k)\,\vec{k}\cdot\vec{E}$ (primer šibkega zunanjega polja $\vec{E}$)

Predpostavka: - energijske ploskve morajo biti krogelno simetrične.
- W odvisna samo od kota $\theta_{kk'}$ med $\vec{k}$ in $\vec{k}'$.
- elastični trki.

$$\frac{g(k)}{\tau(\varepsilon_k)} = \int \frac{d^3\vec{k}'}{(2\pi)^3}\, W(\vec{k},\vec{k}')\,[f(\vec{k}) - f(\vec{k}')]$$

$$= \int \frac{d^3\vec{k}'}{(2\pi)^3}\, W(\vec{k},\vec{k}')\,A(k)\,(\vec{k}-\vec{k}')\cdot\vec{E}$$

$\vec{k}\parallel z$ $\quad$ $\vec{E}$ polarni kot θ $\quad$ $\vec{k}'$ polarni kot θ', azimut ϕ'

$$\vec{k}\cdot\vec{E} = kE\cos\theta \qquad\qquad |\vec{k}| = |\vec{k}'|$$

$$\vec{k}\cdot\vec{k}' = kk'\cos\theta'$$

$$\vec{k}'\cdot\vec{E} = k'E\,(\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos\phi')$$ sferični zakon o kosinusih

$$(\vec{k}-\vec{k}')\cdot\vec{E} = kE\,[\cos\theta\,(1-\cos\theta') - \sin\theta\sin\theta'\cos\phi']$$

odpade ob integraciji po ϕ'

$d\cos\theta'\,d\varphi$

$$\frac{g(k)}{\tau(\varepsilon_k)} = \int \frac{d^3\vec{k}'}{(2\pi)^3}\,(1-\cos\theta')\,W(\vec{k},\vec{k}')\cdot \underbrace{A(k)\,E\,k\cos\theta}_{g(k)}$$

$$\frac{1}{\tau(\varepsilon_k)} = \int \frac{d^3\vec{k}'}{(2\pi)^3}\, W(\vec{k},\vec{k}')\,(1-\cos\theta')$$

Sipanje naprej ($\theta'=0$) ne prispeva!

Če $\tilde{\vec{F}}$ ni odvisna od $\vec{r}$ in če $\vec{B}=0$, lahko rešimo:

$$g(\vec{k}) = -\tau(\varepsilon_k)\frac{\partial f_0}{\partial \varepsilon}\vec{v}_k \cdot \tilde{\vec{F}}$$

4.12. EL. PREVODNOST

$$\vec{F} = q\vec{E}$$

$$\vec{j}_e = \frac{-2q^2}{(2\pi)^3}\int d^3\vec{k}\, \underbrace{\vec{v}_k(\vec{v}_k\cdot\vec{E})}_{(\vec{v}_k\otimes\vec{v}_k)\vec{E}}\frac{\partial f_0}{\partial \varepsilon}\tau(\varepsilon_k)$$

izpostavimo: $\tau(\varepsilon_k)\approx\tau(\varepsilon_F)$.

$$\sigma_{\alpha\beta} = 2q^2\int\frac{d^3\vec{k}}{(2\pi)^3}v_\alpha(\vec{k})\underbrace{v_\beta(\vec{k})\left(-\frac{\partial f_0}{\partial\varepsilon}\right)}_{-\frac{1}{\hbar}\frac{\partial f_0}{\partial k_\beta}}\overline{\tau(\varepsilon_k)}$$

Per partes (na površini BZ velja $\nabla_k\varepsilon_{\vec{k}}=0$):

$$\sigma_{\alpha\beta} = 2q^2\tau(\varepsilon_F)\int\frac{d^3\vec{k}}{(2\pi)^3}f_0(\varepsilon_{\vec{k}})\frac{1}{\hbar}\frac{\partial v_\alpha(\vec{k})}{\partial k_\beta}$$

$$= 2q^2\tau(\varepsilon_F)\int_{\text{zasedena}}\frac{d^3\vec{k}}{(2\pi)^3}\left[M(\vec{k})^{-1}\right]_{\alpha\beta} \qquad \left[M(\vec{k})^{-1}\right]_{\alpha\beta} = \frac{1}{\hbar^2}\frac{\partial^2\varepsilon(\vec{k})}{\partial k_\alpha\partial k_\beta}$$

Prosti elektroni: $M^{-1}_{\alpha\beta} = \frac{\delta_{\alpha\beta}}{m} \Rightarrow \sigma_{\alpha\beta} = \delta_{\alpha\beta}\frac{nq^2\tau(\varepsilon_F)}{m}$ (Drude)

$T=0$

$$\sigma(\varepsilon) = 2q^2\int\frac{d^3\vec{k}}{(2\pi)^3}\delta(\varepsilon-\varepsilon_k)\vec{v}_k\otimes\vec{v}_k\,\tau(\varepsilon_k)$$

5. ~~13~~. TRANSPORTNI KOEFICIENTI

Lokalno ravnovesje:

$\mu = \mu(\vec{r}) \qquad T = T(\vec{r})$

↳ oz. $\eta = \eta(\vec{r})$ → pogoj za ravnovesje je homogen elektrokemijski potencial.

- En faktor $(\varepsilon_k - \mu)$ v definiciji $\vec{j}_Q$.
- En faktor $(\varepsilon_k - \mu)$ v izrazu za $\vec{F}$.

$$\vec{j}_n = L_{11}\, q\vec{\mathcal{E}} + L_{12}\left(-\frac{\nabla T}{T}\right)$$

$$\vec{j}_Q = L_{21}\, q\vec{\mathcal{E}} + L_{22}\left(-\frac{\nabla T}{T}\right)$$

Onsagerjeve zveze: $L_{ij}(B) = L_{ji}(-B)$

(Nobelova iz kemije, 1968)

$L_{11} = K_0 \qquad L_{12} = L_{21} = K_1 \qquad L_{22} = K_2$

$$K_n = 2\int \frac{d^3k}{(2\pi)^3}\left(-\frac{\partial f_0}{\partial \varepsilon_k}\right)\tau(\varepsilon_k)\,(\varepsilon_k - \mu)^n\, \vec{v}_k \otimes \vec{v}_k$$

Izraze za koeficiente izpeljemo za homogena polja. Vseeno veljajo tudi za lokalne transportne enačbe.

Uvedemo lahko "energijsko odvisno prevodnost" $\sigma(\varepsilon)$.

$$K_n = \frac{1}{q^2}\int d\varepsilon\left(-\frac{\partial f_0}{\partial \varepsilon}\right)(\varepsilon - \varepsilon_F)^n\,\sigma(\varepsilon)$$

↗ popravi v μ

$$K_0 = \frac{\sigma(\varepsilon_F)}{q^2} \qquad K_1 = -\frac{(\pi k_B T)^2}{3q^2}\left.\frac{\partial \sigma}{\partial \varepsilon}\right|_{\varepsilon=\varepsilon_F} \qquad K_2 = \frac{(\pi k_B T)^2}{3q^2}\,\sigma(\varepsilon_F)$$

To dobimo z Sommerfeldovim razvojem, $I(T,\varepsilon_F) = \int_{-\infty}^{\infty} d\varepsilon\, f(\varepsilon - \varepsilon_F)\,\phi(\varepsilon) = \frac{\pi D}{\sin(\pi D)}\int_{-\infty}^{\varepsilon_F} d\varepsilon'\,\phi(\varepsilon')$

$D = k_B T \frac{d}{d\varepsilon_F}$

6. ~~14~~. TOPLOTNA PREVODNOST

$$\vec{j}_n = 0 \qquad L_{11}\, q\vec{\mathcal{E}} + L_{12}\left(-\frac{\nabla T}{T}\right) = 0 \qquad q\vec{\mathcal{E}} = L_{11}^{-1} L_{12}\frac{\nabla T}{T}$$

$$\vec{j}_Q = \left(L_{21}L_{11}^{-1}L_{12} - L_{22}\right)\frac{\nabla T}{T} = -\lambda \nabla T$$

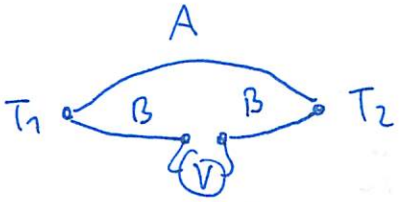
$$\lambda = \left(L_{22} - L_{21}L_{11}^{-1}L_{12}\right)\frac{1}{T} \qquad \lambda \simeq \frac{\pi^2 k_B^2 T}{3q^2}\,\sigma$$

↑ dominantni prispevek v kovinah

(Wiedemann–Franzov zakon)

7 ~~18~~. SEEBECKOV POJAV

$\vec{j}_n = 0 \qquad \vec{E} = Q \cdot \nabla T \qquad Q = \frac{1}{eT} L_{11}^{-1} L_{12}$ Seebeckov koeficient (thermoelectric power)



$$V = \int \vec{E} \cdot d\vec{s} = \int_1^2 Q_A \nabla T \, ds + \int_2^1 Q_B \nabla T \, ds = \underbrace{(Q_B - Q_A)}_{\text{koeficient termonapetosti}} (T_2 - T_1)$$

Neodvisno od poti, neodvisno od temperature pri voltmetru.
En spoj pri referenčni temperaturi, drugi na točki merjenja.
Standardni materiali, tabelirano. $\Rightarrow$ termočlen
Merimo razliko. Za superprevodnike je $Q = 0$, omogoča absolutne meritve.

NI BISTVENO

$Q \approx -\frac{\pi^2 k_B^2 T}{3 e \sigma} \frac{\partial \sigma}{\partial \varepsilon}\Big|_{\varepsilon = \varepsilon_F}$ (Mottova formula)

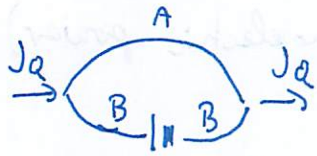
$\frac{T_{RT}}{T_F} \sim \frac{1}{100}$ ker $\frac{T_{RT}}{1\,eV} \sim \frac{1}{40}$

$\frac{k_B}{e} \approx 86\,\mu V/K \qquad \frac{1}{\sigma} \frac{\partial \sigma}{\partial \varepsilon} \sim \frac{1}{\varepsilon_F} \qquad |Q| \sim 1\,\mu V/K$ tipičen velikostni red

Občutljivo na predznak nosilcev naboja in na asimetrijo med delci in vrzelmi v bližini Fermijevega nivoja, ter na sipalne procese. Težko zanesljivo izračunati. Strog test teorije.

8.16. PELTIEROV POJAV

Naj velja $T_1 = T_2$ $(\nabla T = 0)$.



$$\left.\begin{aligned} \tilde{j}_e &= q^2 L_{11} \tilde{\mathcal{E}} \\ \tilde{j}_Q &= q L_{21} \tilde{\mathcal{E}} \end{aligned}\right\} \quad \tilde{j}_Q = \frac{L_{21}}{q L_{11}} \tilde{j}_e = \Pi \tilde{j}_e$$

$$\Pi = \frac{L_{21}}{q L_{11}} \quad \text{Peltierov koeficient}$$

$$J_Q = J_{Q,A} + J_{Q,B} = (\Pi_A - \Pi_B) j_e$$

toplotna tokova se seštejeta v končno količino

Uporabno za hlajenje brez mehanskih delov.

DO TU, 26.3.2024 (dodamo: izvor entropije)

9.17. MAGNETOUPORNOST IN HALLOV POJAV

Linearizirana Boltzmannova + približek relaksacijskega časa:

$$\frac{\partial f_0}{\partial \varepsilon}\vec v_k\cdot\vec E + \frac{1}{\hbar}\frac{\partial g}{\partial \vec k}\cdot \vec v_k\times\vec B + \frac{g}{e\tau} = 0$$

Naj velja $\vec v_k = \frac{1}{\hbar}\frac{\partial \varepsilon_k}{\partial k} = \frac{\hbar \vec k}{m^*} \Rightarrow \frac{\partial v_i}{\partial k_j} = \frac{\hbar}{m^*}\delta_{ij}$

Nastavek: $g = -e\tau \frac{\partial f_0}{\partial\varepsilon}\vec v_k\cdot\vec X$

$$\vec v_k\cdot\vec E = \vec v_k\cdot\vec X + \frac{e\tau}{m^*}\vec X\cdot(\vec v_k\times\vec B)$$

$$\vec E = \vec X + \frac{e\tau}{m^*}\vec B\times\vec X$$

$$\vec j_e = \frac{2e}{(2\pi)^3}\int d^3k\, \vec v_k \left(e\tau \vec v_k\cdot\vec X\right)\left(-\frac{\partial f_0}{\partial\varepsilon}\right)$$

predpostavimo izotropen v_k in $\tau(\varepsilon_k)$. Potem je $\underline{\underline{\sigma}} = \sigma\, \underline{\underline{1}}$.

$$\vec j_e = \sigma\vec X \iff \vec X = \rho\, \vec j_e$$

$$\vec E = \rho\,\vec j_e + \frac{\rho e\tau}{m^*}\vec B\times\vec j_e$$

$\vec B = B\hat e_z$

$x \Rightarrow E_\parallel = \rho\, j_e$

$y \Rightarrow E_\perp = \frac{\rho e\tau}{m^*} B\, j_e$ ← Hallov pojav

Če sta $v(\varepsilon_k)$ ali $\tau(\varepsilon_k)$ anizotropna, bo tudi longitudinalna komponenta el. polja odvisna od B. To je magnetoupornost. Majhen učinek. Tipično $\propto B^2$.

POVEŠ NA KONCU

GMR: v heterostrukturah iz magnetnih in nemagnetnih materialov. Omogočilo revolucijo v gostoti podatkov na magnetnih medijih. Nobelova Fert, Grünberg (2007)

TMR: tunelski spoj med dvema feromagnetnima plastema. Se uporablja v MRAM in sodobnih bralnih glavah v trdih diskih.

CMR: v manganovih oksidih, gre za fazni prehod med prevodnim in izolatorskim stanjem. Ni uporabno v praksi.

Pozor

$$\rho_0 = \frac{m}{n e^2 \tau}$$

$$\vec{E} = \underline{\underline{\rho}}\, \vec{j} \qquad \underline{\underline{\rho}} = \rho_0 \begin{pmatrix} 1 & \omega_c \tau & 0 \\ -\omega_c \tau & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\omega_c = \frac{eB}{m}$$ ciklotronska frekvenca

$$\Delta \rho_{xx}(B) = \rho_{xx}(B) - \rho_{xx}(0) \rightarrow$$ tukaj nič za izotropen primer

$$\underline{\underline{\sigma}} = \underline{\underline{\rho}}^{-1} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$$

$$\sigma_{xx}(B) = \rho_0^{-1} \frac{1}{1+(\omega_c \tau)^2} = \sigma_{yy}(B) \qquad \sigma_{zz}(B) = \rho_0^{-1}$$

$$-\sigma_{xy}(B) = +\sigma_{yx}(B) = \rho_0^{-1} \frac{\omega_c \tau}{1+(\omega_c \tau)^2}$$

Če k prevodnosti prispeva več pasov, dobimo magnetoupornost tudi če je sistem izotropen. Posledica tega, da se seštevajo tenzorji prevodnosti, zato je $\underline{\underline{\rho}}^{-1} = \underline{\underline{\rho}}_1^{-1} + \underline{\underline{\rho}}_2^{-1}$.

IZVOR ENTROPIJE

$$T dS = dU - \mu dN \qquad dV = 0,\ V = \text{konst}$$

$$S = V\rho s \qquad U = V\rho u \qquad N = Vn$$

$$\rho T ds = \rho du - \mu dn$$

Gostota entropijskega toka: $\vec{j}_S = \frac{\vec{j}_Q}{T}$

$\vec{j}_S = \frac{1}{T}(\vec{j}_u - \mu \vec{j}_n)$

Kontinuitetna: $\frac{\partial n}{\partial t} + \nabla \cdot \vec{j}_n = 0$ (ohranitev št. delcev)

$$\rho \frac{\partial u}{\partial t} + \nabla \cdot \vec{j}_u = \vec{F} \cdot \vec{j}_n \qquad \text{z} \quad \vec{F} = q\vec{E}$$

NI POTREBNO V IZPELJAVI Σ z $\vec{j}_n$ in $\vec{j}_Q$

Gostota totalne energije (notranja energija + potencial zunanjega polja):

$$\rho u_{tot} = \rho u + q n \varphi(\vec{r})$$

Gostota totalnega energijskega toka:

$$\vec{j}_{u,tot} = \vec{j}_u + q \vec{j}_n \varphi(\vec{r})$$

$$\rho \frac{\partial u_{tot}}{\partial t} + \nabla \cdot \vec{j}_{u,tot} = \underbrace{\rho \frac{\partial u}{\partial t} + \nabla \cdot \vec{j}_u} + q\frac{\partial n}{\partial t} \varphi + \nabla \cdot (q \vec{j}_n \varphi)$$

$$= \cancel{q\vec{E} \cdot \vec{j}_n} + \cancel{q\varphi \frac{\partial n}{\partial t}} + \cancel{q\varphi \nabla \cdot \vec{j}_n} + \cancel{q \vec{j}_n \cdot \underbrace{\nabla \varphi}_{-\vec{E}}}$$

$$\rho \frac{\partial u_{tot}}{\partial t} + \nabla \cdot \vec{j}_{u,tot} = 0 \qquad \text{(ohranitev energije)}$$

KONTINUITETNA ENAČBA ZA ENTROPIJO:

$$\rho \frac{\partial s}{\partial t} + \nabla \cdot \vec{j}_s = \underbrace{\frac{1}{T} \rho \frac{\partial u}{\partial t}} - \cancel{\frac{1}{T}\mu \frac{\partial n}{\partial t}} - \frac{\nabla T}{T^2}(\vec{j}_u - \mu \vec{j}_n) + \underbrace{\frac{1}{T} \nabla \cdot \vec{j}_u} - \cancel{\frac{\mu}{T} \nabla \cdot \vec{j}_n} - \frac{1}{T} \vec{j}_n \cdot \nabla \mu$$

$$= \frac{1}{T} \vec{F} \cdot \vec{j}_n - \frac{\nabla T}{T^2}(\vec{j}_u - \mu \vec{j}_n) - \frac{1}{T} \nabla \mu \cdot \vec{j}_n$$

skip $\left[= \frac{1}{T} \left[\vec{j}_n \cdot \left(\vec{F} - T\nabla\left(\frac{\mu}{T}\right)\right) + \vec{j}_u \cdot \left(-\frac{\nabla T}{T}\right)\right] \right]$

$q\vec{\mathcal{E}} = q\vec{E} - \nabla \mu$

$$= \frac{1}{T} \left[\vec{j}_n \cdot (\underbrace{\vec{F} - \nabla \mu}_{q\vec{\mathcal{E}}}) + \vec{j}_Q \cdot \left(-\frac{\nabla T}{T}\right) \right] = \Sigma$$